Definition 1. A ***latin square*** is a $n \times n$ matrix over some set, $\mathfrak{S}$, such that $|\mathfrak{S}| = n$ and all elements of $\mathfrak{S}$ occur in each row and each column of the matrix exactly once.

Definition 2. A 4-tuple, (a, b, c, d) of elements of a matrix M is said to be a ***quadrangle*** if it is of the form $(m_{i,j}, m_{i,k}, m_{l,k}, m_{l,j})$. That is, if the four elements are the corners of a rectangular block in M, with at least two rows and two columns, such that a and c lie on one of the diagonals of the rectangular block.

Definition 3. A matrix M is said to satisfy the ***quadrangle criterion*** if whenever (a, b, c, d) and (a', b', c', d') are two quadrangles satisfying $a = a'$, $b = b'$, and $c = c'$; then $d = d'$.

Theorem 1. *Let M be a latin square of order n. Then M is the multiplication table of a finite group (of order n) if and only if the quadrangle criterion holds for M.*

Proof. Let $(m_{i,j}, m_{i,k}, m_{l,k}, m_{l,j})$ and $(m_{i',j'}, m_{i',k'}, m_{l',k'}, m_{l',j'})$ be two quadrangles from M such that $m_{i,j} = m_{i',j'}$, $m_{i,k} = m_{i',k'}$, and $m_{l,k} = m_{l',k'}$. Then:

$$\begin{aligned}
m_{l,j} &= m_l m_j \\
&= m_l(m_k m_k^{-1})(m_i^{-1} m_i) m_j \\
&= (m_l m_k)(m_i m_k)^{-1}(m_i m_j) \\
&= m_{l,k} m_{i,k}^{-1} m_{i,j} \\
&= m_{l',k'} m_{i',k'}^{-1} m_{i',j'} \\
&= (m_{l'} m_{k'})(m_{i'} m_{k'})^{-1}(m_{i'} m_{j'}) \\
&= m_{l'}(m_{k'} m_{k'}^{-1})(m_{i'}^{-1} m_{i'}) m_{j'} \\
&= m_{l'} m_{j'} \\
&= m_{l',j'}
\end{aligned}$$

Conversely, we will show that if the quadrangle criterion holds for M, then M is a Cayley table of a finite group, G.
For M to be a group table, we need to pick a header and a sideline. WOLOG, we pick the first row and first column, respectively. Then $m_{1,1}$ will be the identity element of G, e. Since M is a latin square, e occurs once in each row and column, so $m_i x = e$ and $y m_j = e$ are soluble for every choice of m_i and m_j.
Let a, b, and c be arbitrary elements from M. If one of them is equal to e, then $a(bc) = (ab)c$ is trivial. So, we can assume that none of them are equal to e. Consider the following two portions of M:

	b	bc
e	b	bc
a	ab	$a(bc)$

	e	c
b	b	bc
ab	ab	$(ab)c$

By the quadrangle criterion, $a(bc) = (ab)c$ and so M is the multiplication table of some finite group. □